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**EIGHTH EDITION**

# ENGINEERING MATHEMATICS

**JOHN BIRD**

# Engineering Mathematics

*Eighth Edition*

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John Bird

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# Engineering Mathematics

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## Why is knowledge of mathematics important in engineering?

A career in any engineering or scientific field will require both basic and advanced mathematics. Without mathematics to determine principles, calculate dimensions and limits, explore variations, prove concepts and so on, there would be no mobile telephones, televisions, stereo systems, video games, microwave ovens, computers or virtually anything electronic. There would be no bridges, tunnels, roads, skyscrapers, automobiles, ships, planes, rockets or most things mechanical. There would be no metals beyond the common ones, such as iron and copper, no plastics, no synthetics. In fact, society would most certainly be less advanced without the use of mathematics throughout the centuries and into the future.

*Electrical engineers* require mathematics to design, develop, test, or supervise the manufacturing and installation of electrical equipment, components, or systems for commercial, industrial, military or scientific use.

*Mechanical engineers* require mathematics to perform engineering duties in planning and designing tools, engines, machines, and other mechanically functioning equipment; they oversee installation, operation, maintenance and repair of such equipment as centralised heat, gas, water and steam systems.

*Aerospace engineers* require mathematics to perform a variety of engineering work in designing, constructing, and testing aircraft, missiles and spacecraft; they conduct basic and applied research to evaluate adaptability of materials and equipment to aircraft design and manufacture and recommend improvements in testing equipment and techniques.

*Nuclear engineers* require mathematics to conduct research on nuclear engineering problems or apply principles and theory of nuclear science to problems concerned with release, control and utilisation of nuclear energy and nuclear waste disposal.

*Petroleum engineers* require mathematics to devise methods to improve oil and gas well production and determine the need for new or modified tool designs; they oversee drilling and offer technical advice to achieve economical and satisfactory progress.

*Industrial engineers* require mathematics to design, develop, test, and evaluate integrated systems for managing industrial production processes, including human work factors, quality control, inventory control, logistics and material flow, cost analysis and production coordination.

*Environmental engineers* require mathematics to design, plan or perform engineering duties in the prevention, control and remediation of environmental health hazards, using various engineering disciplines; their work may include waste treatment, site remediation or pollution control technology.

*Civil engineers* require mathematics in all levels in civil engineering - structural engineering, hydraulics and geotechnical engineering are all fields that employ mathematical tools such as differential equations, tensor analysis, field theory, numerical methods and operations research.

Knowledge of mathematics is therefore needed by each of the engineering disciplines listed above.

It is intended that this text -*Engineering Mathematics* - will provide a step by step approach to learning fundamental mathematics needed for your engineering studies.

Now in its eighth edition, *Engineering Mathematics* is an established textbook that has helped thousands of students to succeed in their exams. John Bird's approach is based on worked examples and interactive problems. Mathematical theories are explained in a straightforward manner, being supported by practical engineering examples and applications in order to ensure that readers can relate theory to practice. The extensive and thorough topic coverage makes this an ideal text for a range of Level 2 and 3 engineering courses. This title is supported by a companion website with resources for both students and lecturers, including lists of essential formulae and multiple choice tests.

**John Bird**, BSc (Hons), CEng, CMath, CSci, FIMA, FIET, FCollT, is the

former Head of Applied Electronics in the Faculty of Technology at Highbury College, Portsmouth, UK. More recently, he has combined freelance lecturing at the University of Portsmouth with examiner responsibilities for Advanced Mathematics with City and Guilds, and examining for the International Baccalaureate Organisation. He is the author of some 130 textbooks on engineering and mathematical subjects with worldwide sales of over one million copies. He is a chartered engineer, a chartered mathematician, a chartered scientist and a Fellow of three professional institutions, and is currently lecturing at the Defence School of Marine and Air Engineering in the Defence College of Technical Training at HMS Sultan, Gosport, Hampshire, UK.

## Preface

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*Engineering Mathematics, 8th Edition* covers a wide range of syllabus requirements. In particular, the book is suitable for any course involving engineering mathematics and in particular for the latest **National Certificate and Diploma courses and City & Guilds syllabuses in Engineering**.

This text will provide a foundation in mathematical principles, which will enable students to solve mathematical, scientific and associated engineering principles. In addition, the material will provide engineering applications and mathematical principles necessary for advancement onto a range of Incorporated Engineer degree profiles. It is widely recognised that a students' ability to use mathematics is a key element in determining subsequent success. First year undergraduates who need some remedial mathematics will also find this book meets their needs.

In *Engineering Mathematics, 8th Edition*, **new material** is included on metric conversions, metric to imperial conversions, numbering systems, convergence, Bayes theorem, accuracy of numerical methods, Maclaurin's series, together with other minor modifications and chapter re-ordering.

Throughout the text, theory is introduced in each chapter by an outline of essential definitions, formulae, laws and procedures. The theory is kept to a minimum, for **problem solving** is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

For clarity, the text is divided into **eleven topic areas**, these being: number and algebra, areas and volumes, trigonometry, graphs, complex numbers, vectors, statistics, differential calculus, integral calculus, differential equations and further number and algebra.

This new edition covers, in particular, the following syllabi:

- i. **Mathematics for Technicians**, the core unit for **National Certificate/Diploma** courses in Engineering, to include all or part of

the following chapters:

1. **Algebraic methods:** 2,5,11,13,14,28,30(1, 4, 6, 8, 9 and 10 for revision)
  2. **Trigonometric methods and areas and volumes:** 18-20, 22-25, 33, 34
  3. **Statistical methods:** 37, 38
  4. **Elementary calculus:** 45, 52, 59
- ii. **Further Mathematics for Technicians**, the optional unit for **National Certificate/Diploma** courses in Engineering, to include all or part of the following chapters:
1. **Advanced graphical techniques:** 29-31
  2. **Algebraic techniques:** 15,33,37,38
  3. **Trigonometry:** 22-27
  4. **Calculus:** 45-47, 52, 58-60
- iii. **Mathematics contents of City & Guilds Technician Certificate/Diploma courses**
- iv. Any **introductory/access/foundation course** involving Engineering Mathematics at **University, Colleges of Further and Higher Education and in schools.**

Each topic considered in the text is presented in a way that assumes in the reader little previous knowledge of that topic.

*Engineering Mathematics, 8th Edition* provides a follow-up to *Basic Engineering Mathematics, 7th Edition* and a lead into *Higher Engineering Mathematics, 8th Edition* .

This textbook contains over **1000 worked problems**, followed by some **1850 further problems** (all **with answers at the back of the book**). The further problems are contained within some **243 practice exercises**; each Exercise follows on directly from the relevant section of work, every two or three pages. In addition, the text contains **243 multiple-choice questions**. Where at all possible, the problems mirror practical situations found in engineering and science. **571 line diagrams** enhance the understanding of the theory.

At regular intervals throughout the text are some **19 Revision Tests** to check understanding. For example, Revision Test 1 covers material contained in



Chapters 1 to 4, Revision Test 2 covers the material in Chapters 5 to 8 and so on. These Revision Tests do not have answers given since it is envisaged that lecturers could set the tests for students to attempt as part of their course structure. Lecturers' may obtain a set of solutions of the Revision Tests in an **Instructor's Manual** available via the internet - see below.

A list of **essential formulae** is included in the text for convenience of reference.

'**Learning by Example**' is at the heart of *Engineering Mathematics, 8th Edition*.

**JOHN BIRD**

**Royal Naval Defence College of Marine and Air Engineering, HMS Sultan, formerly of University of Portsmouth and Highbury College, Portsmouth**

**Free Web downloads at <http://www.routledge.com/cw/bird>**

#### **For students**

1. **Full solutions** to the 1850 questions contained in the 243 Practice Exercises
2. Download **multiple choice questions and answer sheet**
3. **List of essential formulae**
4. **Famous engineers/scientists** - 25 are mentioned in the text

#### **For instructors/lecturers**

1. **Full solutions** to the 1850 questions contained in the 243 Practice Exercises
2. **Full solutions** and marking scheme to each of the **19 revision tests** - named as **Instructors guide**
3. **Revision tests** - available to run off to be given to students
4. **Download multiple choice questions and answer sheet**
5. **List of essential formulae**
6. **Illustrations** - all 571 available on PowerPoint
7. **Famous engineers/scientists** - 25 are mentioned in the text

## Section 1

# Number and algebra

## Revision of fractions, decimals and percentages

### *Why it is important to understand: Revision of fractions, decimals and percentages*

Engineers use fractions all the time, examples including stress to strain ratios in mechanical engineering, chemical concentration ratios and reaction rates, and ratios in electrical equations to solve for current and voltage. Fractions are also used everywhere in science, from radioactive decay rates to statistical analysis. Also, engineers and scientists use decimal numbers all the time in calculations. Calculators are able to handle calculations with fractions and decimals; however, there will be times when a quick calculation involving addition, subtraction, multiplication and division of fractions and decimals is needed. Engineers and scientists also use percentages a lot in calculations; for example, percentage change is commonly used in engineering, statistics, physics, finance, chemistry and economics. When you feel able to do calculations with basic arithmetic, fractions, decimals and percentages, with or without the aid of a calculator, then suddenly mathematics doesn't seem quite so difficult.

**At the end of this chapter you should be able to:**

- add, subtract, multiply and divide with fractions
- understand practical examples involving ratio and proportion
- add, subtract, multiply and divide with decimals
- understand and use percentages

## 1.1 Fractions

When 2 is divided by 3, it may be written as  $\frac{2}{3}$  or  $2 \div 3$ .  $\frac{2}{3}$  is called a **fraction**. The number above the line, i.e. 2, is called the **numerator** and the number below the line, i.e. 3, is called the **denominator**.

When the value of the numerator is less than the value of the denominator, the fraction is called a **proper fraction**; thus  $\frac{2}{3}$  is a proper fraction. When the value of the numerator is greater than the denominator, the fraction is called an **improper fraction**. Thus  $\frac{7}{3}$  is an improper fraction and can also be expressed as a **mixed number**, that is, an integer and a proper fraction. Thus the improper fraction  $\frac{7}{3}$  is equal to the mixed number  $2 \frac{1}{3}$ .

When a fraction is simplified by dividing the numerator and denominator by the same number, the process is called **cancelling**. Cancelling by 0 is not permissible.

**Problem 1.** Simplify:  $\frac{1}{3} + \frac{2}{7}$

The lowest common multiple (i.e. LCM) of the two denominators is  $3 \times 7$ , i.e. 21

Expressing each fraction so that their denominators are 21, gives:

$$\frac{1}{3} + \frac{2}{7} = \frac{1 \times 7}{3 \times 7} + \frac{2 \times 3}{7 \times 3} = \frac{7}{21} + \frac{6}{21} = \frac{7 + 6}{21} = \frac{13}{21}$$

Alternatively:

$$\frac{1}{3} + \frac{2}{7} = \text{Step (2)} \downarrow \downarrow \text{Step (3)} \downarrow \downarrow \frac{(7 \times 1)}{21} + \frac{(3 \times 2)}{21} \uparrow \text{Step (1)}$$

Step1: the LCM of the two denominators;

Step2: for the fraction  $\frac{1}{3}$ , 3 into 21 goes 7 times,  $7 \times$  the numerator is  $7 \times 1$ ;

Step3: for the fraction  $\frac{2}{7}$ , 7 into 21 goes 3 times,  $3 \times$  the numerator is  $3 \times 2$

Thus  $\frac{1}{3} + \frac{2}{7} = \frac{7 + 6}{21} = \frac{13}{21}$  as obtained previously.

**Problem 2.** Find the value of  $\frac{3}{2} - \frac{2}{3} - \frac{1}{6}$

One method is to split the mixed numbers into integers and their fractional parts. Then

$$3\frac{2}{3} - 2\frac{1}{6} = 3 + \frac{2}{3} - 2 + \frac{1}{6} = 3 + \frac{2}{3} - 2 - \frac{1}{6} = 1 + \frac{4}{6} - \frac{1}{6} = 1\frac{3}{6} = 1\frac{1}{2}$$

Another method is to express the mixed numbers as improper fractions.

$$\text{Since } 3 = \frac{9}{3}, \text{ then } 3\frac{2}{3} = \frac{9}{3} + \frac{2}{3} = \frac{11}{3}$$

$$\text{Similarly, } 2\frac{1}{6} = \frac{12}{6} + \frac{1}{6} = \frac{13}{6}$$

Thus  $3\frac{2}{3} - 2\frac{1}{6} = \frac{11}{3} - \frac{13}{6} = \frac{22}{6} - \frac{13}{6} = \frac{9}{6} = 1\frac{1}{2}$  as obtained previously.

**Problem 3.** Determine the value of

$$4\frac{5}{8} - 3\frac{1}{4} + 1\frac{2}{5}$$

$$4\frac{5}{8} - 3\frac{1}{4} + 1\frac{2}{5} = (4 - 3 + 1) + \frac{5}{8} - \frac{1}{4} + \frac{2}{5} = 2 + \frac{5}{8} - \frac{2}{8} + \frac{2}{5} = 2 + \frac{3}{8} + \frac{2}{5} = 2 + \frac{15}{40} + \frac{16}{40} = 2\frac{31}{40}$$

**Problem 4.** Find the value of  $3\frac{7}{14} \times 14\frac{15}{15}$

Dividing numerator and denominator by 3 gives:

Dividing numerator and denominator by 7 gives:

This process of dividing both the numerator and denominator of a fraction by the same factor(s) is called **cancelling**.

**Problem 5.** Evaluate:  $1\frac{3}{5} \times 2\frac{1}{3} \times 3\frac{3}{7}$

Mixed numbers **must** be expressed as improper fractions before multiplication can be performed. Thus,

$$1\frac{3}{5} \times 2\frac{1}{3} \times 3\frac{3}{7} = \frac{5}{5} + \frac{3}{5} \times \frac{6}{3} + 1\frac{3}{7} + \frac{3}{7}$$

**Problem 6.** Simplify:  $3\frac{7}{12} \div 2\frac{21}{21}$

$$3\frac{7}{12} \div 2\frac{21}{21} = 3\frac{7}{12} \div 2$$

Multiplying both numerator and denominator by the reciprocal of the denominator gives:

This method can be remembered by the rule: invert the second fraction and change the operation from division to multiplication. Thus:

as obtained previously.

**Problem 7.** Find the value of  $5\frac{3}{5} \div 7\frac{1}{3}$

The mixed numbers must be expressed as improper fractions. Thus,

**Problem 8.** Simplify:

$$1\frac{3}{4} - 2\frac{5}{8} + 1\frac{4}{3} \div 3\frac{8}{13} \times 1\frac{3}{4}$$

The order of precedence of operations for problems containing fractions is the same as that for integers, i.e. remembered by **BODMAS** (**B**rackets, **O**f, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction). Thus,

$$1\frac{3}{4} - 2\frac{5}{8} + 1\frac{4}{3} \div 3\frac{8}{13} \times 1\frac{3}{4}$$

**Problem 9.** Determine the value of

$$7\frac{6}{11} \text{ of } 3\frac{1}{2} - 2\frac{1}{4} + 5\frac{1}{8} \div 3\frac{1}{16} - 1\frac{2}{3}$$

**Now try the following Practice Exercise**

### Practice Exercise 1 Fractions (Answers on page 672)

Evaluate the following:

- (a)  $1\frac{2}{3} + 2\frac{5}{8}$  (b)  $7\frac{1}{6} - 1\frac{4}{5}$
- (a)  $2\frac{7}{11} + 3\frac{1}{11}$  (b)  $2\frac{9}{11} - 1\frac{7}{11} + 2\frac{3}{11}$
- (a)  $10\frac{3}{7} - 8\frac{2}{3}$  (b)  $3\frac{1}{4} - 4\frac{4}{5} + 1\frac{5}{6}$
- (a)  $3\frac{4}{5} \times 5\frac{9}{10}$  (b)  $17\frac{3}{5} \times 15\frac{1}{19}$
- (a)  $3\frac{5}{9} \times 7\frac{9}{10} \times 1\frac{2}{7}$  (b)  $13\frac{1}{7} \times 4\frac{7}{11} \times 3\frac{4}{39}$
- (a)  $3\frac{8}{9} \div 4\frac{5}{64}$  (b)  $1\frac{1}{3} \div 2\frac{5}{9}$
- $1\frac{2}{3} + 3\frac{5}{8} \div 8\frac{1}{15} - 1\frac{3}{4}$
- $7\frac{1}{15}$  of  $15 \times 5\frac{7}{8} + 3\frac{4}{5} \div 15\frac{1}{16}$
- $1\frac{4}{5} \times 2\frac{3}{4} - 1\frac{3}{4} \div 3\frac{5}{7} + 2\frac{7}{8}$
- $2\frac{3}{4} \times 1\frac{1}{4} \div 2\frac{3}{4} + 1\frac{4}{5} + 1\frac{3}{5}$
- If a storage tank is holding 450 litres when it is three-quarters full, how much will it contain when it is two-thirds full?
- Three people, P, Q and R contribute to a fund. P provides  $\frac{3}{5}$  of the total, Q provides  $\frac{2}{3}$  of the remainder, and R provides £8. Determine (a) the total of the fund, (b) the contributions of P and Q.